

Geometry from Information Geometry*

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Abstract

We use the method of maximum entropy to model physical space as a curved statistical manifold. It is then natural to use information geometry to explain the geometry of space. We find that the resultant information metric does not describe the full geometry of space but only its conformal geometry — the geometry up to local changes of scale. Remarkably, this is precisely what is needed to model “physical” space in general relativity.

1 Introduction

The motivation behind the program of Entropic Dynamics is the realization that the formulation of physical theories makes essential use of concepts that are clearly designed for processing information. Prominent examples include probability, entropy, and — as we will argue in this work — geometry. This suggests that the connection between physics and nature is somewhat indirect: the goal of physics is not to provide a direct and faithful image of nature but to provide a framework for processing information and making inferences [1]–[5].

This view imposes severe restrictions on physical models because the tools and methods of physics must inevitably reflect their inferential origins. Probabilities, for example, must necessarily be epistemic; they are, after all, the tools that have been designed to quantify uncertainty. The entropies must be information entropies; they are tools for updating or assigning probabilities. What could perhaps be most surprising is that even the geometries that pervade physics might also be of statistical origin; it might be possible to explain them in terms of information geometry [2][6].

As with any application of entropic methods, Entropic Dynamics requires that we specify the subject matter (the microstates) and the relevant information (the constraints) on the basis of which we will carry out our inferences. In this paper we take the first step towards formulating an entropic dynamics of gravity — we identify the subject matter. (Two other relevant contributions in this direction are [7][8].)

*Presented at MaxEnt 2015, the 35th International Workshop on Bayesian Inference and Maximum Entropy Methods in Science and Engineering (July 19–24, 2015, Potsdam NY, USA).

We use the method of maximum entropy to model “physical” three dimensional space as a curved statistical manifold. The basic idea is that the points of space are not defined with perfect resolution; they are not structureless dots. When we say that a particle is located at a point x , its actual true location x' is uncertain and lies somewhere in the vicinity of x . Thus, to each point x in space one associates a probability distribution, $p(x'|x)$. In this model space is a statistical manifold and is automatically endowed with an information metric. It is important to emphasize that information geometry yields positive definite metrics which apply to Riemannian manifolds. The problem of modelling the pseudo-Riemannian geometry of spacetime remains open.

We find that the resultant information geometry does not specify the full geometry of the statistical manifold. It allows an arbitrary choice of the local scale which means that what is described is the conformal geometry of space. Remarkably, this is precisely what is needed to model “physical” space in general relativity. Indeed, there is convincing evidence that the dynamical degrees of freedom of the gravitational field represent the conformal geometry of space-like hypersurfaces embedded in space-time [9]-[12].

The construction is straightforward except for one technical difficulty. Since coordinates do not themselves carry any information — we can always change coordinates — it is essential to maintain covariance under coordinate transformations. The difficulty is that the expected value constraints required by the method of maximum entropy do not transform covariantly. The problem is overcome by applying the method of maximum entropy in the flat tangent spaces and using the exponential map to induce probabilities on the manifold itself.

In our brief closing remarks we observe that when space is modeled as a statistical manifold it acquires a curious hybrid character and exhibits features that are typical of discrete spaces while maintaining the calculational advantages of continuous manifolds. Finally, we show that the information volume of a region of space is a measure of the effective number of distinguishable points and is also a measure of its entropy.

2 Distinguishability and information distance

Our subject is space which we model as a smooth three-dimensional manifold \mathbf{X} . There are no external rulers and therefore there is no externally imposed notion of distance. The main assumption is that \mathbf{X} is blurred; its points are defined with a finite resolution. Consider a test particle (or a field variable) located at $x \in \mathbf{X}$ with coordinates x^a , $a = 1, 2, 3$. When we say that the test particle is at x it turns out that it is actually located at some unknown x' somewhere in its vicinity. The uncertainty in x' is represented by a probability distribution, $p(x'|x)$. (The probability that x' lies within d^3x' is $p(x'|x)d^3x'$.) We need not, at this point, specify the physical origin of the uncertainty. Since to each point $x \in \mathbf{X}$ one associates a probability distribution $p(x'|x)$ the space \mathbf{X} is a special type of statistical manifold.

In a generic statistical manifold \mathbf{M} one associates a probability distribution $p(\xi|x)$ to each point $x \in \mathbf{M}$. The variables ξ and x need not represent physical quantities of the same kind. For example, in the case of a gas ξ can represent the positions and momenta of the molecules, while x could stand for the temperature and volume of the gas. Here we deal with a special type of statistical manifold in which both x' and x are positions in the same space.

Coordinates x^a are introduced to distinguish one point from another, but if points are blurred we cannot fully distinguish the point at x^a from another point at $x^a + dx^a$. We seek a quantitative measure of the extent to which these two distributions can be distinguished. It is remarkable that this measure is determined uniquely by imposing certain symmetries that are natural for statistical manifolds — the invariance under Markovian embeddings. It is even more remarkable that this unique measure has all the properties of a distance [2][6]. Such *information distance* is given by

$$d\ell^2 = g_{ab}(x) dx^a dx^b, \quad (1)$$

where the metric tensor g_{ab} — *the information metric* — is given by,

$$g_{ab}(x) = \int dx' p(x'|x) \partial_a \log p(x'|x) \partial_b \log p(x'|x). \quad (2)$$

(We adopt the standard notation $\partial_a = \partial/\partial x^a$ and $dx' = d^3x'$.)

To complete the definition of the information geometry of \mathbf{X} we must specify a connection or covariant derivative ∇ . This allows us to introduce notions of parallel transport, curvature, and so on. Although we will not use covariant derivatives in this work, it appears that the Levi-Civita connection, defined so that $\nabla_a g_{bc} = 0$, is the candidate of choice. It is the simplest among all the α -connections [6], and it is also the most natural because it does not require that any additional structure be imposed on the Hilbert space of functions $(p)^{1/2}$ [13].

Thus the space \mathbf{X} inherits its geometry from the family of distributions $p(x'|x)$. Even at this early stage we can envision potentially important consequences for later applications to quantum gravity. Contrary to naive expectation the statistical manifolds proposed here are not rougher, more irregular, than those needed to describe classical gravity. In fact they may be considerably smoother because irregularities at scales smaller than the local uncertainty become meaningless.

3 Using maximum entropy to assign $p(x'|x)$

Next we use the method of maximum entropy to assign the distribution $p(x'|x)$. The central physical assumption is that the physically relevant information that is necessary to model space is captured by constraints on the expectation of x' and of its uncertainty. Therefore one is led to consider the expected value

$$\langle x'^a \rangle = \int dx' p(x'|x) x'^a, \quad (3)$$

and the variance-covariance matrix

$$\langle (x'^a - x^a)(x'^b - x^b) \rangle = \int dx' p(x'|x) (x'^a - x^a)(x'^b - x^b) . \quad (4)$$

The problem with covariance A problem arises immediately because in a curved space neither of these constraints is covariant. To see the difficulty consider a change of coordinates. Let $x^i = X^i(x^a)$ and $x'^i = X^i(x'^a)$ where the indices abc and ijk denote old and new coordinates respectively. Taylor expand x'^i in $(x'^a - x^a)$,

$$x'^i - x^i = X_a^i (x'^a - x^a) + \frac{1}{2} X_{ab}^i (x'^a - x^a)(x'^b - x^b) + \dots \quad (5)$$

where

$$X_a^i = \partial_a X^i(x) = \frac{\partial x^i}{\partial x^a} \quad \text{and} \quad X_{ab}^i = \partial_a \partial_b X^i(x) = \frac{\partial^2 x^i}{\partial x^a \partial x^b} . \quad (6)$$

Taking the expected value with the scalar $dx' p(x'|x)$ gives

$$\langle x'^i - x^i \rangle = X_a^i \langle x'^a - x^a \rangle + \frac{1}{2} X_{ab}^i \langle (x'^a - x^a)(x'^b - x^b) \rangle + \dots \quad (7)$$

which shows that we can impose $\langle x'^a \rangle = x^a$, but on changing coordinates we will have

$$\langle x'^i \rangle = x^i + O(\sigma^2) \quad (8)$$

where $O(\sigma^2)$ represents a non-negligible correction of the order of the width of the distribution. Therefore neither coordinate differences $x'^a - x^a$, eq.(5), nor their expected values, eq.(7), transform as components of a vector.

The Exponential map The problem with the noncovariance of expected values can be traced to the fact that the statistical manifold \mathbf{X} is curved. To evade this problem the entropy maximization will be carried out in the flat spaces \mathbf{T}_P that are tangent to \mathbf{X} at each point P and then a special map — the *exponential map* — will be used to obtain the corresponding induced distributions on \mathbf{X} .

The exponential map is defined as follows. Assume that the manifold \mathbf{X} has a metric g_{ab} — we shall later see that it does. Then we can construct geodesics. Consider the space \mathbf{T}_P that is tangent to \mathbf{X} at P . Each vector $\vec{y} \in \mathbf{T}_P$ defines a unique geodesic through P . Let the points $Q \in \mathbf{X}$ on the geodesic through P with tangent vector \vec{y} be denoted by $Q(\vec{y}; \lambda)$ where λ is an affine parameter such that $Q(\vec{y}; \lambda = 0) = P$. Then the point with affine parameter $\lambda = 1$ is assigned coordinates y^i ,

$$Q^i(\vec{y}; \lambda = 1) = y^i . \quad (9)$$

This construction maps a straight line in the flat tangent space \mathbf{T}_P to a straight line in the curved manifold \mathbf{X} . The set of vectors $\lambda \vec{y} \in \mathbf{T}_P$ is mapped to the set of points $x^i = \lambda y^i \in \mathbf{X}$. The map $\mathbf{T}_P \rightarrow \mathbf{X}$ is called *the exponential map*

and the corresponding coordinates have several useful properties [14]. For our purposes we will only need

$$g_{ij}(P) = \delta_{ij} \quad \text{and} \quad \partial_k g_{ij}(P) = 0 . \quad (10)$$

These coordinates are called Riemann Normal Coordinates at P (denoted NC_P).¹

Using MaxEnt on the tangent spaces Our goal is to assign the distribution $p(Q|P) = p(x'|x)$ on the basis of information about the expected position and its uncertainty. This information is provided through constraints defined on the tangent space \mathbf{T}_P . We use the method of maximum entropy to assign a distribution $\hat{p}(\vec{y}|P)$ on \mathbf{T}_P and the exponential map is then used to induce the corresponding distribution $p(x'|x)$ on \mathbf{X} .

Consider a point $P \in \mathbf{X}$ with generic coordinates x^a and a positive definite tensor field $\gamma^{ab}(x)$. The components of a vector $\vec{y} \in \mathbf{T}_P$ are y^a . The distribution $\hat{p}(\vec{y}|P)$ is assigned on the basis of information about the expected location on \mathbf{T}_P ,

$$\langle y^a \rangle_P = \int d^3y \hat{p}(y|P) y^a = 0 , \quad (11)$$

and the variance-covariance matrix

$$\langle y^a y^b \rangle_P = \int d^3y \hat{p}(y|P) y^a y^b = \gamma^{ab}(P) . \quad (12)$$

It is always possible to transform to new coordinates

$$x^i = X^i(x^a) \quad \text{such that} \quad \gamma^{ij}(P) = \delta^{ij} \quad \text{and} \quad \partial_k \gamma^{ij}(P) = 0 . \quad (13)$$

(If γ^{ab} where a metric tensor this would be a transformation to NC_P .) The new components of \vec{y} are

$$y^i = X_a^i y^a \quad \text{where} \quad X_a^i = \partial_a X^i(x) = \frac{\partial x^i}{\partial x^a} , \quad (14)$$

and the constraints (11) and 12 take a simpler form,

$$\langle y^i \rangle_P = 0 \quad \text{and} \quad \langle y^i y^j \rangle_P = \delta^{ij} . \quad (15)$$

The distribution we seek is that which maximizes the entropy

$$S[\hat{p}, q] = - \int dy \hat{p}(\vec{y}|P) \log \frac{\hat{p}(\vec{y}|P)}{q(\vec{y})} \quad (16)$$

¹The exponential map from \mathbf{T}_P to \mathbf{X} is 1-1 only within some neighborhood of P — the so-called *normal* neighborhood. Beyond this neighborhood the geodesics in a curved manifold may cross. In such cases the mapping remains well defined but it no longer serves the useful purpose of defining a coordinate system. For the smooth statistical manifolds that interest us we can expect that the normal neighborhoods will extend over rather large regions which renders the exponential map particularly useful.

relative to a measure $q(\vec{y})$. Since \mathbf{T}_P is flat we take $q(\vec{y})$ to be constant and we may ignore it. Maximizing $S[\hat{p}, q]$ subject to the constraints (15) and normalization yields

$$\hat{p}(\vec{y}|P) = \exp \left[-\alpha - \beta_i y^i - \frac{1}{2} \gamma_{ij} y^i y^j \right] , \quad (17)$$

where α , β_i , and γ_{ij} are Lagrange multipliers. Requiring that $\hat{p}(\vec{y}|P)$ satisfy the constraints implies that $e^{-\alpha}$ is just a normalization constant, that the three multipliers β_i vanish, and that the matrix γ_{ij} is the inverse of the covariance matrix $\gamma^{ij} = \delta^{ij}$, that is, $\gamma_{ij} \gamma^{jk} = \delta_i^k$. Therefore $\gamma_{ij} = \delta_{ij}$ and

$$\hat{p}(\vec{y}|P) = \frac{(\det \gamma_{ij})^{1/2}}{(2\pi)^{3/2}} \exp \left[-\frac{1}{2} \gamma_{ij} y^i y^j \right] = \frac{1}{(2\pi)^{3/2}} \exp \left[-\frac{1}{2} \delta_{ij} y^i y^j \right] . \quad (18)$$

We can now transform back to the original coordinates y^a using the inverse of eq.(14),

$$y^a = X_i^a y^i \quad \text{and} \quad \gamma_{ab} = X_a^i X_b^j \delta_{ij} . \quad (19)$$

The resulting distribution is also Gaussian,

$$\hat{p}(\vec{y}|P) = \frac{(\det \gamma_{ab})^{1/2}}{(2\pi)^{3/2}} \exp \left[-\frac{1}{2} \gamma_{ab} y^a y^b \right] . \quad (20)$$

Next we use an exponential map to induce the corresponding distribution $p(x'|x)$ on the manifold \mathbf{X} . We use the tensor γ^{ab} as if it were the inverse of a metric tensor. This allows us to define the corresponding “geodesics” and exponential map. Let the coordinates of the point $P \in \mathbf{X}$ be denoted $x^i(P)$, then the normal coordinates of neighboring points will be denoted

$$x'^i = x^i(P) + y^i , \quad (21)$$

and the distribution $p(x'|P) = p(x'^i|x^i)$ induced by $\hat{p}(\vec{y}|P)$, eq.(18), is

$$\begin{aligned} p(x'^i|x^i) &= \frac{(\det \gamma_{ij})^{1/2}}{(2\pi)^{3/2}} \exp \left[-\frac{1}{2} \gamma_{ij} (x'^i - x^i)(x'^j - x^j) \right] \\ &= \frac{1}{(2\pi)^{3/2}} \exp \left[-\frac{1}{2} \delta_{ij} (x'^i - x^i)(x'^j - x^j) \right] . \end{aligned} \quad (22)$$

In NC_P the distribution (22) retains the Gaussian form, just like (18). We can now transform back to the generic frame x^a of coordinates and define $p(x'^a|x^a)$ by

$$p(x'^a|x^a) d^3 x'^a = p(x'^i|x^i) d^3 x'^i , \quad (23)$$

which is a covariant identity between scalars and holds in all coordinate systems. In the x^a coordinates the distribution $p(x'^a|x^a)$ will not, in general, be Gaussian,

$$p(x'^a|x^a) = \frac{(\det \gamma_{ab})^{1/2}}{(2\pi)^{3/2}} \exp \left[-\frac{1}{2} \delta_{ij} (X^i(x'^a) - X^i(x^a)) (X^j(x'^a) - X^j(x^a)) \right] . \quad (24)$$

4 The information geometry of space

Next we calculate the information metric g_{ab} associated with the distributions (24). The direct substitution of eq.(24) into eq.(2) yields an integral that can be handled by transforming to NC_P . Using eq.(23) and $\partial_a = X_a^i \partial_i$, we get

$$g_{ab}(x) = X_a^i X_b^j \int d^3 x'^i p(x'^i | x^i) \partial_i \log p(x'^i | x^i) \partial_j \log p(x'^i | x^i) . \quad (25)$$

Since $p(x'^i | x^i)$ is Gaussian, eq.(22), this integral is straightforward. First substitute eq.(21), then integrate using eq.(15), and finally transform back to the original generic coordinates x^a using eq.(19), to get

$$g_{ab} = X_a^i X_b^j \delta_{ij} = \gamma_{ab} . \quad (26)$$

This result is deceptively simple: *the information metric g_{ab} is the inverse of the covariance tensor γ^{ab} that describes the blurriness of space.* But one should not let formal simplicity stand in the way of appreciating its significance. The metric (26) represents a potentially fruitful conceptual development. The idea might best be conveyed through a historical analogy. The concept of temperature was first introduced as an unexplained “degree of hotness”. Temperature was operationally defined as whatever was measured by peculiar devices called thermometers and eventually it came to be interpreted as an average kinetic energy per degree of freedom. It took a long time before arriving at the modern entropic interpretation of temperature T as a Lagrange multiplier ($\beta = 1/kT$) in a maximum entropy distribution. It is conceivable that the notion of distance might undergo a similar development. Distance has long been taken for granted — an unexplained quantity measured by peculiar devices called rulers. The main result of this paper is to suggest that the metric of space g_{ab} is a statistical concept that measures a “degree of distinguishability” and that it can be traced to Lagrange multipliers γ_{ab} that describe the blurriness γ^{ab} of space .

5 Discussion and conclusions

Canonical quantization of gravity? From the perspective of information geometry any attempt to quantize gravity by imposing commutation relations on the metric tensor g_{ab} , that is, on the Lagrange multipliers γ_{ab} , is misguided — it leads to a dead end. It would appear to be just as misguided as attempting to formulate a quantum theory of fluids by imposing commutation relations on those Lagrange multipliers (like temperature, pressure, or chemical potential) that define the statistical macrostate.

Dimensionless distance? There is one very peculiar feature of the information distance $d\ell$ in eq.(1) that turns out to be very significant: $d\ell$ is dimensionless. Indeed, we can easily verify in eq.(2) that if dx^i has units of length, then $p(x'|x)$ has units of inverse volume, and g_{ij} has units of inverse length squared.

Distances are supposed to be measured in units of length; what sort of distance is this $d\ell$?

A simple example will help clarify this issue. Consider two neighboring Gaussian distributions,

$$p(x'|x) = \frac{(\det \gamma_{ij})^{1/2}}{(2\pi)^{3/2}} \exp \left[-\frac{1}{2} \gamma_{ij} (x'^i - x^i)(x'^j - x^j) \right] \quad (27)$$

and $p(x'|x + dx)$, with means x and $x + dx$ and the same covariance matrix, $\gamma_{ij} = \delta_{ij}/\sigma^2$. The distance between them, eq.(1), is

$$d\ell^2 = \frac{1}{\sigma^2} \delta_{ij} dx^i dx^j . \quad (28)$$

This is the Euclidean metric δ_{ij} rescaled by σ^2 . Therefore the dimensionless $d\ell$ represents a distance measured in units of the uncertainty σ . More generally, *the information metric $g_{ij}(x)$ measures distinguishability in units of the local uncertainty implied by the distribution $p(x'|x)$.*

As long as we are concerned with quantifying *distinguishability* no global unit of length is needed, but if what we want is a measure of absolute *distance* several related questions arise: How do we compare the uncertainties at two different locations? Or, alternatively, does the local uncertainty provide us with a universal, global standard of length? If it does not, and the uncertainty varies from point to point, how do we compare lengths at different places?

The answer is that an absolute comparison of how the uncertainty varies from point to point is objectively meaningless because there are no external rulers. Information geometry does not provide comparisons of lengths at different locations but it does allow us to compare short lengths at the same location. This means that *what the information geometry describes is the conformal geometry of space*. It describes the local “shape” of space but not its absolute local “size”.

Nevertheless, we humans can still adopt some criterion that determines a local scale and allows us to define length as a tool for reasoning, as an aid for constructing pictures and models of the world. In such models geometry is described by a metric tensor \bar{g}_{ab} that is conformally related to the information metric in (26),

$$\bar{g}_{ab}(x) = \sigma^2(x) g_{ab}(x) . \quad (29)$$

The choice of the scale factor $\sigma^2(x)$, which amounts to a choice of “gauge”, is a matter of convenience. It is dictated by purely pragmatic considerations: *length is defined so that physics looks simple*. The scale of distance — just like the duration of time — turns out to be a property not of the world but of the models we employ to describe it. One possible choice of gauge would be, for example, to legislate that $\sigma^2(x) = \sigma_0^2$ is a constant. Another possibility is to choose $\sigma^2(x)$ so that the evolving three-dimensional manifold \mathbf{X} generates a four-dimensional space-time.²

²The conditions for such a *space-time gauge* involve scale factors that satisfy the Lichnerowicz-York equation [9][10]. Similar notions have also been proposed in the context of Machian relational dynamics by Barbour and his collaborators. (See *e.g.* [11][12].)

The statistical state of space The state of space is the joint distribution of all the y_x variables associated to every point x . We assume that the y_x variables at x are independent of the $y_{x'}$ variables at x' , and therefore their joint distribution is a product,

$$\hat{P}[y|g] = \prod_x \hat{p}(y_x|x, g_x) , \quad (30)$$

where g_x is short for $g_{ab}(x)$. Given $g_{ab}(x)$ at any point x the distribution $\hat{p}(y_x|x, g_x)$ in the tangent space \mathbf{T}_x is Gaussian,

$$\hat{p}(y_x|x, g_x) = \frac{(\det g_x)^{1/2}}{(2\pi)^{3/2}} \exp \left[-\frac{1}{2} g_{ab}(x) y_x^a y_x^b \right] . \quad (31)$$

We conclude that the information metric g_{ab} determines the statistical state of space.

Continuous and/or discrete? A perhaps unexpected consequence of the notion of an information distance is the following. Suppose we want to measure the size of a finite region of space by counting the number of points within it. Counting points depends on a decision what we mean by a point and, in particular, on what we mean by two different points. If we agree that two points ought to be counted separately only when we can distinguish them then one can assert that the number of distinguishable points in any finite region is finite. Therefore, the answer to the old question of whether space is continuous or discrete is that, in a certain sense, it is both. If the local uncertainty at x is described as $\sigma(x)$ then, roughly, a volume contains one distinguishable point per volume $\frac{4}{3}\pi\sigma^3(x)$ and a surface contains one distinguishable point per area $\pi\sigma^2(x)$. Remarkably this allows us to compare the sizes of regions of different dimensionality: it is meaningful to assert that a surface and a volume are of the same (information) size whenever they contain the same number of distinguishable points. Furthermore, since the number of distinguishable points within d^3x is $g^{1/2}d^3x$, this suggests that sums over distinguishable points can be given a continuum representation by replacing sums by integrals,

$$\sum_x (\dots) \rightarrow \int d^3x g^{1/2} (\dots) . \quad (32)$$

It is also to be expected that modelling space as a statistical manifold will provide a natural regulator that will eliminate the divergences that normally afflict relativistic quantum field theories.

The entropy of space As an example we calculate the total entropy of space,

$$S[\hat{P}, \hat{Q}] = - \int Dy \hat{P}[y|g] \log \frac{\hat{P}[y|g]}{\hat{Q}[y|g]} \stackrel{\text{def}}{=} S[g] \quad (33)$$

relative to the uniform distribution

$$\hat{Q}[y|g] = \prod_x g^{1/2}(x) , \quad (34)$$

which is independent of y — a constant. Since the y 's in eq.(30) are independent variables the entropy is additive, $S[g] = \sum_x S(x)$, and we need to calculate the entropy $S(x)$ associated to a point at a generic location x ,

$$S(x) = - \int d^3y \hat{p}(y|x, g_x) \log \frac{\hat{p}(y|x, g_x)}{g^{1/2}(x)} = \frac{3}{2} \log 2\pi e = s_0 . \quad (35)$$

Thus, the entropy per point is a numerical constant ($s_0 \approx 4.2568$) and the entropy of any region R of space, $S_R[g]$, is just its information volume,

$$S_R[g] = \sum_{x \in R} S(x) = s_0 \int_R d^3x g^{1/2}(x) . \quad (36)$$

Thus, the information volume $\int_R d^3x g^{1/2}$ of a region R is a measure of the effective number of distinguishable points within it and also a measure of its entropy.

Summary Physical space is modelled as a statistical manifold \mathbf{X} — to each point $x \in \mathbf{X}$ one associates a probability distribution $p(x'|x)$. This automatically endows the space \mathbf{X} with a geometry — an information geometry that determines its conformal geometry.

The problem of assigning $p(x'|x)$ in a way that guarantees covariance is addressed by focusing attention on vector variables y_x that live in each tangent space \mathbf{T}_x . The method of maximum entropy is used to assign the distributions $\hat{p}(y_x|x)$ at each x and then the exponential map $\mathbf{T}_P \rightarrow \mathbf{X}$ is used to induce the corresponding distributions $p(x'|x)$ on \mathbf{X} . The validity of the construction rests on the assumption that the normal neighborhood of every point x — the region about x where the exponential map is 1-1 — is sufficiently large. The assumption is motivated by the intuition that the statistical manifolds \mathbf{X} are very smooth. Indeed, when points are separated by distances less than the local uncertainty σ they cannot be effectively distinguished and it is not possible to have curvatures larger than $1/\sigma$.

Acknowledgments I would like to thank D. Bartolomeo, C. Cafaro, N. Caticha, S. DiFranzo, A. Giffin, P. Goyal, S. Ipek, D.T. Johnson, K. Knuth, S. Nawaz, M. Reginatto, C. Rodríguez, J. Skilling, and C.-Y. Tseng for many discussions on entropy, inference, and information geometry.

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